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A MINIMUM BOUNDARY CONDITION ERROR ALGORITHM FOR THIN WIRE RADI--ETC(U)
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Phase Report
August 1977

A MINIMUM BOUNDARY CONDITION ERROR ALGORITHM FOR THIN WIRE RADIATION AND SCATTERING PROBLEMS

Syracuse University

ROME AIR DEVELOPMENT CENTER
Air Force Systems Command
Griffiss Air Force Base, New York 13441

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APPROVED:

KENNETH R. SIARKIEWICZ
Project Engineer

APPROVED:

JOSEPH J. NARESKY
Chief, Reliability & Compatibility Division

FOR THE COMMANDER:

JOHN P. HUSS
Acting Chief, Plans Office

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This report contains results of numerical tests on a method of moments algorithm for the solution of thin wire radiation and scattering problems. The algorithm has convergence behavior similar to a highly overdetermined field point matching scheme. It has been assumed, therefore, that information about the range of applicability of overdetermined wire formulations can be inferred from the results of these tests. Unfortunately, in the tests performed so far, the algorithm has proven to be generally inferior to the other formulations tested. (Con't)

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The report is divided into three sections. The algorithm, called MBCRE (for Minimum Boundary Condition Residual Error) is described in Section I. Section II explains the test result tables and also the connection between the MBCRE algorithm and overdetermined point matching schemes. Section III is a listing of the computer program used for the tests.

INTRODUCTION

This report contains results of numerical tests on a method of moments algorithm for the solution of thin wire radiation and scattering problems. The algorithm has convergence behavior similar to a highly overdetermined field point matching scheme. It has been assumed, therefore, that information about the range of applicability of overdetermined wire formulations can be inferred from the results of these tests. Unfortunately, in the tests performed so far, the algorithm has proven to be generally inferior to the other formulations tested.

The report is divided into three sections. The algorithm, called MBCRE (for Minimum Boundary Condition Residual Error) is described in Section I. Section II explains the test result tables and also the connection between the MBCRE algorithm and overdetermined point matching schemes. Section III is a listing of the computer program used for the tests.

Section I.

The integral equation relating the current distribution on a thin wire to the tangential component of some arbitrary impressed electromagnetic field is

$$E_z(z) = \frac{j}{\omega\epsilon}\left(\kappa^2 + \frac{\partial^2}{\partial z^2}\right)\int_0^{\ell} G(z,z')I(z')dz' \tag{1}$$

$$G(z,z') = \frac{e^{-j\kappa((z-z')^2 + a^2)^{1/2}}}{((z-z')^2 + a^2)^{1/2}}$$

where z and z' are coordinates of distance along the wire; $E_z(z)$ is the known tangential component of the impressed electric field along the cylindrical wire surface; $I(z')$ is the (unknown) current along the wire axis; ℓ is the wire length; ϵ,μ are the material parameters (electric permittivity and magnetic permeability) of the medium in which the wire is imbedded; ω is the radian frequency; a is the wire radius; and k ($= \omega\sqrt{\mu\epsilon}$) is the wave number.

The algorithm discussed in this report for the solution of (1) assumes a common moments approximation $I_a(z')$ to the axial current $I(z')$ of the form

$$I_a(z') = \sum_{i=1}^{N} I_i f_i(z') \tag{2}$$

where the $f_i(z')$ are piecewise sinusoidal expansion functions given by

$$f_i(z') = \begin{cases} \dfrac{\sin \kappa(z' - z_1)}{\sin \kappa(z_2 - z_1)} & z_1 < z' < z_2 \\ \dfrac{\sin \kappa(z_3 - z')}{\sin \kappa(z_3 - z_2)} & z_2 < z' < z_3 \\ 0 & \text{elsewhere} \end{cases} \tag{3}$$

and the I_i are a set of unknown amplitudes. (See Fig. 1)

When the approximation $I_a(z')$ is inserted in (1) it will produce a tangential field along the wire differing from the forcing function $E_z(z)$ by a residual error $r(z)$,

$$r(z) = E_z(z) - \frac{j}{\omega\varepsilon}\left(\kappa^2 + \frac{\partial^2}{\partial z^2}\right) \int_0^{\ell} G(z,z') I_a(z') dz' \tag{4}$$

It is desired to find the set of I_i for which the integrated mean square error is minimum, i.e.

$$e_T = \int_0^{\ell} r^*(z) r(z) dz \qquad (*) \text{ denotes complex conjugate} \tag{5}$$

From equations (3), (4) and (5) it can be seen that e_T is a positive definite real quadratic function of the I_i with a unique minimum at the point where:

$$\frac{\partial e_T}{\partial x_k} = 0 \qquad \frac{\partial e_T}{\partial y_k} = 0 \tag{6}$$

$$I_k = x_k + jy_k$$

Applying condition (6) to equation (5) gives:

$$\int_0^{\ell} \left(r^*(z) \frac{\partial r(z)}{\partial x_k} + \frac{\partial r^*(z)}{\partial x_k} r(z)\right) dz = 0 \tag{7a}$$

$$\int_0^{\ell} \left(r^*(z) \frac{\partial r(z)}{\partial y_k} + \frac{\partial r^*(z)}{\partial y_k} r(z)\right)dz = 0 \tag{7b}$$

$k = 1,2,\ldots.\ N.$

The derivatives in (7) have the form

$$\frac{\partial r(z)}{\partial x_k} = -h_k(z) \qquad \frac{\partial r^*(z)}{\partial x_k} = -h_k^*(z) \tag{8}$$

$$\frac{\partial r(z)}{\partial y_k} = -jh_k(z) \qquad \frac{\partial r^*(z)}{\partial y_k} = jh_k^*(z)$$

$$h_k(z) = \frac{j}{\omega\varepsilon}\left(\kappa^2 + \frac{\partial^2}{\partial z^2}\right) \int_0^{\ell} G(z,z')f_k(z')dz'$$

Inserting these back in (7) and simplifying the result gives

$$-2\ \mathrm{Re}\left[\int_0^{\ell} r(z)h_k^*(z)dz\right] = 0$$

$$2j\ \mathrm{Im}\left[\int_0^{\ell} r(z)h_k^*(z)dz\right] = 0$$

or

$$\int_0^{\ell} \left(E_z(z) - \sum_{i=1}^{N} I_i h_i(z)\right)h_k^*(z)dz = 0 \tag{9a}$$

which is an inhomogeneous set of linear equations

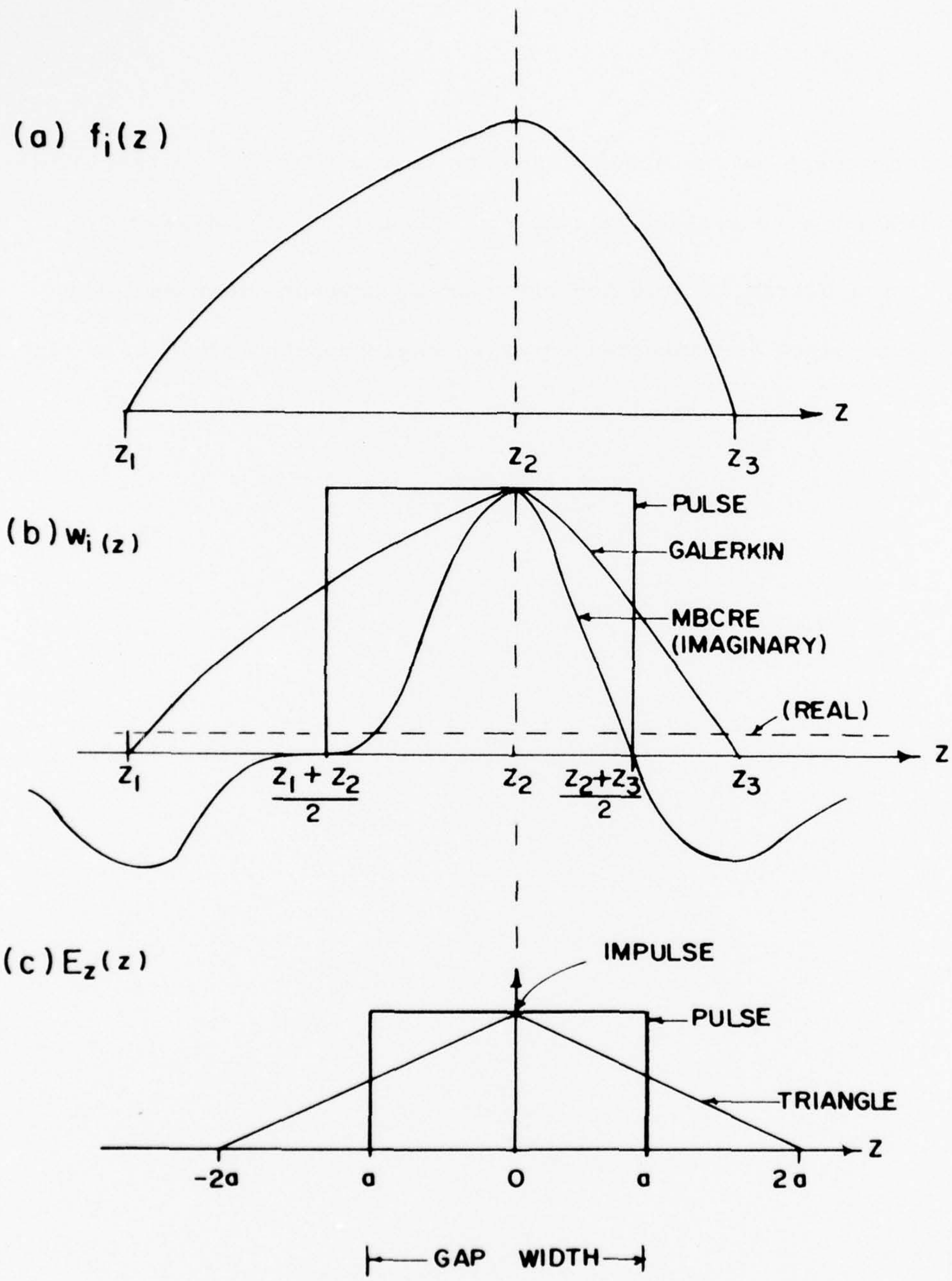
$$v_k = \sum_{i=1}^{N} z_{ki} I_i \qquad \underline{V} = [Z]\underline{I} \tag{9b}$$

$$v_k = \int_0^{\ell} E_z(z)h_k^*(z)dz \qquad z_{ki} = \int_0^{\ell} h_i(z)h_k^*(z)dz$$

with solution

$$\underline{I} = [Z]^{-1}\underline{V} \tag{10}$$

Numerical tests on the above algorithm were confined to cases involving a single straight wire excited across a narrow gap. The extension to collections of arbitrarily oriented skew wires, however, is reasonably straightforward since the integral equation analogous to (1) is of a similar form.



(a) $f_i(z)$
z_1
z_2
z_3
z
(b) $w_i(z)$
PULSE
GALERKIN
MBCRE
(IMAGINARY)
(REAL)
z_1
$\frac{z_1+z_2}{2}$
z_2
$\frac{z_2+z_3}{2}$
z_3
z
(c) $E_z(z)$
IMPULSE
PULSE
TRIANGLE
-2a
a
0
a
2a
z
GAP WIDTH

FIG. 1

Section II

In order to make a comparison between the MBCRE algorithm and other MOM solutions to Equation (1), it is convenient to rewrite (1) in operator form.

$$E_z(z) = L(I) \tag{11a}$$

where the linear operator L is:

$$L(\;) = \frac{j}{\omega\varepsilon}\left(k^2 + \frac{\partial^2}{\partial z^2}\right)\int_0^{\ell} G(z,z')[\quad]dz' \tag{11b}$$

If the inner product of any two functions along the wire is defined as

$$(f,g) = \int_0^{\ell} f(z)g(z)dz \tag{12}$$

then the error e_T of equation (5) becomes

$$e_T = (r^*,r) \tag{13a}$$

with

$$r = E_z - L(I_a) = E_z - \sum_{i=1}^{N} I_i L(f_i) \tag{13b}$$

The generated set of linear equations becomes

$$(h_k^*,E_z) = \sum_{i=1}^{N} I_i (h_k^*, h_i) \qquad k = 1,2, \ldots N \tag{14a}$$

or

$$(L^*(f_k),E_z - \sum_{i=1}^{N} I_i L(f_i)) = (L^*(f_k),r) = 0 \tag{14b}$$

Consider now an arbitrary MOM solution to (1). For a given set of $f_i(z)$, the solution is completely determined by the choice of a set of M linearly independent weighting functions $w_k(z)$. The generated set of linear equations becomes

$$(\omega_k, E_z) = (\omega_k, \sum_{i=1}^{N} I_i L(f_i)) \quad k = 1,2,\ldots M \tag{15a}$$

or, using (13b)

$$(\omega_k, r) = 0 \tag{15b}$$

If the set of equations (15b) is overdetermined, it has no solution, but a unique pseudo-solution can be obtained by defining a discrete residual error vector $\underline{R}$ and a positive definite error function $e_T^D(\underline{R})$ as

$$r_k = (\omega_k, r)/(\omega_k^*, \omega_k)^{1/2} \qquad r_k \in \underline{R} \tag{16a}$$

$$e_T^D = \sum_{k=1}^{M} r_k^* r_k \tag{16b}$$

The minimization of e_T^D over the set of I_i is analogous to the minimization of e_T in Section I. Define

$$I_j = x_j + jy_j \qquad v_k = (\omega_k, E_z)$$

$$z_{kj} = (\omega_k, L(f_j))$$

Then

$$r_k = \frac{1}{(\omega_k^*, \omega_k)^{1/2}} (v_k - \sum_{i=1}^{N} z_{ki} I_i)$$

The condition for a minimum becomes

$$\frac{\partial e_T^D}{\partial x_j} = - \sum_{k=1}^{M} \frac{1}{(\omega_k^*,\omega_k)^{1/2}} (r_k(z_{kj}^*) + r_k^*(z_{kj}))$$

$$= -2 \operatorname{Re}[\sum_{k=1}^{M} \frac{1}{(\omega_k^*,\omega_k)^{1/2}} (r_k(z_{kj}^*))] = 0$$

(17a)

$$\frac{\partial e_T^D}{\partial y_j} = -j \sum_{k=1}^{M} \frac{1}{(\omega_k^*,\omega_k)^{1/2}} (r_k(z_{kj}^*) - r_k^*(z_{kj}))$$

$$= 2j \operatorname{Im}[\sum_{k=1}^{M} \frac{1}{(\omega_k^*,\omega_k)^{1/2}} (r_k(z_{kj}^*))]$$

Therefore

$$\sum_{k=1}^{M} \frac{1}{(\omega_k^*,\omega_k)^{1/2}} r_k z_{kj}^* = 0 \qquad j = 1, 2, \ldots N \tag{17b}$$

In matrix form this can be written as

$$[Z]^H[W]^{-1}\underline{V} = [Z]^H[W]^{-1}[Z]\underline{I} \tag{18a}$$

which has the solution

$$\underline{I} = [[Z]^H[W]^{-1}[Z]]^{-1}[Z]^H[W]^{-1}\underline{V} \tag{18b}$$

Here $[Z]^H$ is the conjugate transpose of $[Z]$ and $[W]$ is the $M\times M$ diagonal matrix whose elements are given by

$$[W]_{kk} = (\omega_k^*,\omega_k)$$

In this way any moments scheme can be developed from an error minimization process. Furthermore, any two schemes which minimize error functions with the same minimum point are equivalent.

If the set of weighting functions is taken to be a contiguous set of unit amplitude pulses of equal width covering the wire, the inner products of (16a) can be approximated by

$$r_k = \frac{1}{(\omega_k^*,\omega_k)^{1/2}} \int_{z_k-\Delta z/2}^{z_k+\Delta z/2} r(z)dz \simeq r(z_k)\frac{\Delta z}{(\Delta z)^{1/2}} \tag{19}$$

where z_k is the coordinate of the center of the kth pulse and Δz is the pulse width. The error function e_T^D becomes

$$e_T^D = \sum_{k=1}^{M} r_k^* r_k = \sum_{k=1}^{M} r^*(z_k)r(z_k)\,\Delta z\left(\frac{\Delta z}{\Delta z}\right) \tag{20}$$

As the set of pulses become infinitely dense, the finite sum passes to the defining integral for e_T

$$\lim_{\Delta z\to 0} e_T^D = \int_0^{\ell} r^*(z)r(z)dz = e_T \tag{21}$$

Thus the MBCRE algorithm is equivalent to a highly overdetermined point matching formulation. Conversely, the criteria of minimum BCRE can be roughly enforced by a variety of overdetermined point matching schemes in a way that is numerically cost competitive with singly determined formulations.

In the following numerical tests, the MBCRE algorithm is compared with two other algorithms (pulse weighting and piecewise-sinusoidal or Galerkin weighting) all using piecewise-sinusoidal expansion functions. The purpose of the tests is to determine whether the MBCRE algorithm gives better results for very sparse or minimal expansion function coverings of thin wires. If the MBCRE algorithm were significantly better under these circumstances, an overdetermined formulation of the type described above

would be less costly to apply to large problems. Unfortunately, the MBCRE algorithm proved to be generally inferior to the Galerkin formulation, and often worse than the pulse formulation.

Explanation of the Tables

The following set of test problems all involve a single straight thin wire excited across a narrow gap by a unit voltage source. For each test problem, a single set of expansion functions $f_i(z)$ was chosen, along with three different sets of weighting functions $\omega_k(z)$ and three different functional models of the excitation field due to the voltage source. The three weighting function sets (pulse, Galerkin, MBCRE) were defined in a manner shown in Figure (1b). The MBCRE weighting functions are (from 14b) the set of conjugate fields of each of the $f_i(z)$. The three gap field models were a unit impulse located at the feed point, a one volt pulse centered at the feed point with a width of the "gap width" of the table, and a one volt isosceles triangle centered at the feed point with a width of twice the gap width as shown in Figure (1c). For each test problem, sets of amplitudes I_i were calculated for all possible combinations of weighting functions and field models, making a total of nine amplitude sets for each set of expansion functions. The expansion functions are listed with their three defining points z_1, z_2, and z_3 of Figure (1a).

The tabulated error figures "Datum Relative Error" and "Minimum Possible Error" (CRE_m) are defined below. In order to have a measure of error for each calculation of the current amplitudes I_i it is necessary to calculate an approximate "true solution" $I_T(z)$ for each test problem. This was done by the method of moments using a very dense covering of piecewise sinusoidal expansion functions with Galerkin weighting and using a one volt impulse to model the field in the gap. The "true solution" was verified whenever possible by comparison with measured data (Ref. 1) or with results obtained by other calculations (Ref. 2). Once $I_T(z)$ is found, it is possible to define for each set of I_i a current error function e(z) and a total RMS error E_c by

$$e(z) = I_T(z) - \sum_{i=1}^{N} I_i f_i(z) \tag{22a}$$

$$E_c = (e^*(z), e(z)) \tag{22b}$$

The set of currents I_i^o which minimize E_c are derived in a manner completely analagous with the minimization of e_T^D and are given by the solution of the set of linear equations

$$(e(z), f_i(z)) = 0 \qquad i = 1,2, \ldots, N \tag{23}$$

The error function can now be broken up into two components

$$e_1(z) = I_T(z) - \sum_{i=1}^{N} I_i^o f_i(z) \tag{24a}$$

$$e_2(z) = e(z) - e_1(z) = \sum_{i=1}^{N} I_i^o f_i(z) - \sum_{i=1}^{N} I_i f_i(z) \tag{24b}$$

Because of the orthogonality condition (23):

$$(e_1^*(z),e_2(z)) = (e_1(z),e_2^*(z)) = 0 \tag{25}$$

and it follows that

$$\begin{aligned}(e^*(z),e(z)) &= (e_1^*(z) + e_2^*(z),e_1(z)+e_2(z)) \\ &= (e_1^*(z),e_1(z)) + (e_1^*(z),e_2(z)) + (e_1(z),e_2^*(z)) \\ &+ (e_2(z),e_2^*(z)) \\ &= |e_1(z)|^2 + |e_2(z)|^2\end{aligned} \tag{26}$$

The terms "Datum Relative Error (DRE)" and "Minimum Possible Error (CRE_m)" are defined as

$$CRE_m = \frac{|e_1(z)|}{|I_T(z)|} \tag{27a}$$

$$DRE = \frac{|e_2(z)|}{\left|\sum_{i=1}^{N} I_i^o f_i(z)\right|}$$

The total error was partitioned in this way because CRE_m is an error which is due entirely to the choice of the set of $f_i(z)$ and can not be removed by any solution algorithm. The DRE is the error component which is entirely the fault of the solution algorithm and is therefore a better measure of accuracy than the total RMS error E_c.

Section III

The computer program "MOP" is designed to be run on the Honeywell GECOS time-sharing system in the Fortran CARDIN subsystem. Input data is read from the end of the program and output is written onto a sequential file (with PRMFL designation "02"), as well as on the on-line printer.

The following is a list of input variables and their units. The wire parameters are

WL = wire length (meters)

B = wave number/2π = $1/\lambda$ (meters).

RAD = wire radius (meters)

The solution for the $I_T(z)$ is specified by:

NPS = number of expansion functions (always equally sized and equally spaced)

KPS = the number of the expansion function on which the unit impulse excitation is centered.

The test problem is specified by:

NC = Number of expansion functions

NE = number of nodes needed to define the three expansion function end points and the gap end points

TE(K) (K = 1,NE) = the z coordinate of node k.

TI(K,J) (K = 1,NC, J = 1,3) = z_j (see Eq. 3) for the kth expansion function

KL = node number for one end of the gap (lowest z).

KH = node number for the other end of the gap (highest z)

There is also an input variable KDEL which is the number of the expan-

sion function amplitude to which the calculated input impedance is referred, i.e.

$$Z_{in} = V_{in}/I_{in} = 1./I(KDEL)$$

The calculation of Z_{in} will only be meaningful if the expansion function f_{KDEL} straddles the gap.

All variables are read in at the end of the program in the order and with the formats given below:

WL, B, RAD	(3F10.6)
NPS, KPS, KDEL	(3I4)
NC, NE	(2I4)
TE(K) K = 1,NE	(F10.6) (each node on a separate line)
TI(K,J) (K = 1,NC),(J = 1,3)	(3I4) (The three defining points for each expansion function on one line)
KL,KH	(2I4)

Example:

Assume as a test problem a center excited half wave dipole of radius 0.01λ. Assume also that it is desired to test the three MOM formulations for a set of three equally spaced expansion functions of equal size covering the wire. If we assume an excitation gap width of 0.05λ and a set of 9 expansion functions for $I_T(z)$, the data cards would look like this.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33
		0	.	5	0	0						1	.	0	0	0						0	.	0	1	0						
			9				5				2																					
			3				7																									
		0	.	0	0	0																										
		0	.	1	2	5																										
		0	.	2	2	5																										
		0	.	2	5	0																										
		0	.	2	7	5																										
		0	.	3	7	5																										
		0	.	5	0	0																										
			1				2				4																					
			2				4				6																					
			4				6				7																					
			3				5																									

The program output is written onto the printer and specified permanent file in the following order.

(1) The input impedance calculated from $I_T(z)$

(2) The current amplitudes of $I_T(z)$

(3) The test problem specifications:

a) wire length

b) wave number

c) wire radius

d) the three defining points z_1, z_2, z_3 for each of the speeified expansion functions

e) the end points of the gap.

(4) CRE_m

(5) The input impedance calculated from the I_i^o

(6) The amplitudes I_i^o

For each combination of excitation field model and weighting function the program prints

(7) The input impedance (referred to I_{KDEL})

(8) The DRE

(9) The I_i

Above items 7, 8 and 9 will appear two numbers (L1 and L2) which both vary from one to three and which designate the type of excitation field and weighting functions i.e.

L1 = 1 for pulse weighting

L1 = 2 for Galerkin weighting

L1 = 3 for MBCRE weighting

L2 = 1 for impulse excitation

L2 = 2 for pulse excitation

L2 = 3 for triangle excitation

All printed currents are in amperes in the format

K, REAL (I(K)), AIMAG(I(K)), CMAG (I(K)), PHASE (I(K))

References:

1) King, Mack and Sandler, Arrays of Cylindrical Dipoles. Cambridge University Press, 1968.

2) Harrington and Mautz, "Wires with Arbitrary Excitation," IEEE Trans. on Antennas and Propagation, July 1967.

TEST PROBLEM 1

Wire Length 0.50 Radius 0.015

Feed Point 0.25 Gap Width 0.05

Minimum Possible Error (CRE_m) 0.049

Expansion Functions

No.	z_1	z_2	z_3
1.	0.000	0.083	0.166
2.	0.083	0.166	0.250
3.	0.166	0.250	0.333
4.	0.250	0.333	0.417
5.	0.333	0.417	0.500
6.			
7.			
8.			
9.			
10.			
11.			
12.			

TEST PROBLEM 1

Datum Relative Error

$E_s(z)$ \ $w_i(z)$	Pulse	Piecewise Sinusoid (Galerkin)	MBCRE
Impulse	0.17	0.045	0.10
Pulse	0.17	0.048	0.13
Triangle	0.17	0.049	0.15

TEST PROBLEM 2

Wire Length 0.50 Radius 0.015

Feed Point 0.25 Gap Width 0.125

Minimum Possible Error (CRE_m) 0.076

Expansion Functions

No.	z_1	z_2	z_3
1.	0.000	0.125	0.250
2.	0.125	0.250	0.375
3.	0.250	0.375	0.500
4.			
5.			
6.			
7.			
8.			
9.			
10.			
11.			
12.			

TEST PROBLEM 2

Datum Relative Error

$w_i(z)$ / $E_s(z)$	Pulse	Piecewise Sinusoid (Galerkin)	MBCRE
Impulse	0.29	0.078	0.095
Pulse	0.29	0.070	0.33
Triangle	0.24	0.091	0.35

TEST PROBLEM 3

Wire Length 0.50

Radius 0.015

Feed Point 0.25

Gap Width 0.05

Minimum Possible Error (CRE_m) 0.107

Expansion Functions

No.	z_1	z_2	z_3
1.	0.000	0.200	0.250
2.	0.200	0.250	0.300
3.	0.250	0.300	0.500
4.			
5.			
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11.			
12.			

TEST PROBLEM 3

Datum Relative Error

$E_s(z)$ \ $w_i(z)$	Pulse	Piecewise Sinusoid (Galerkin)	MBCRE
Impulse	0.53	0.13	0.42
Pulse	0.53	0.14	0.46
Triangle	0.67	0.15	0.47

TEST PROBLEM 4

Wire Length 0.50 Radius 0.015

Feed Point 0.250 Gap Width 0.05

Minimum Possible Error (CRE_m) 0.103

Expansion Functions

No.	z_1	z_2	z_3
1.	0.000	0.250	0.500
2.	0.200	0.250	0.300
3.			
4.			
5.			
6.			
7.			
8.			
9.			
10.			
11.			
12.			

TEST PROBLEM 4

Datum Relative Error

$w_i(z)$ / $E_s(z)$	Pulse	Piecewise Sinusoid (Galerkin)	MBCRE
Impulse	0.78	0.13	0.42
Pulse	0.78	0.14	0.46
Triangle	0.77	0.15	0.47

TEST PROBLEM 5

Wire Length 1.000 | Radius 0.015

Feed Point 0.500 | Gap Width 0.100

Minimum Possible Error (CRE_m) 0.08

Expansion Functions

No.	z_1	z_2	z_3
1.	0.000	0.250	0.400
2.	0.250	0.400	0.500
3.	0.400	0.500	0.600
4.	0.500	0.600	0.750
5.	0.600	0.750	1.000
6.			
7.			
8.			
9.			
10.			
11.			
12.			

TEST PROBLEM 5

Datum Relative Error

$E_s(z)$ \ $w_i(z)$	Pulse	Piecewise Sinusoid (Galerkin)	MBCRE
Impulse	0.13	0.058	0.32
Pulse	0.13	0.125	0.25
Triangle	0.22	0.161	0.28

TEST PROBLEM 6

Wire Length 1.0 Radius 0.015

Feed Point 0.50 Gap Width 0.10

Minimum Possible Error (CRE_m) 0.090

Expansion Functions

No.	z_1	z_2	z_3
1.	0.000	0.100	0.200
2.	0.100	0.200	0.300
3.	0.200	0.300	0.400
4.	0.300	0.400	0.500
5.	0.400	0.500	0.600
6.	0.500	0.600	0.700
7.	0.600	0.700	0.800
8.	0.700	0.800	0.900
9.	0.800	0.900	1.000
10.			
11.			
12.			

TEST PROBLEM 6

Datum Relative Error

$E_s(z)$ \ $w_i(z)$	Pulse	Piecewise Sinusoid (Galerkin)	MBCRE
Impulse	0.072	0.020	0.336
Pulse	0.072	0.097	0.179
Triangle	0.165	0.134	0.212

TEST PROBLEM 7

Wire Length 1.0 Radius 0.015

Feed Point 0.50 Gap Width 0.10

Minimum Possible Error (CRE_m) 0.141

Expansion Functions

No.	z_1	z_2	z_3
1.	0.000	0.166	0.333
2.	0.166	0.333	0.500
3.	0.333	0.500	0.666
4.	0.500	0.666	0.833
5.	0.666	0.833	1.000
6.			
7.			
8.			
9.			
10.			
11.			
12.			

TEST PROBLEM 7

Datum Relative Error

$E_s(z)$ \ $w_i(z)$	Pulse	Piecewise Sinusoid (Galerkin)	MBCRE
Impulse	0.092	0.045	0.59
Pulse	0.092	0.070	0.18
Triangle	0.109	0.105	0.22

TEST PROBLEM 8

Wire Length 1.0 Radius 0.015

Feed Point 0.50 Gap Width 0.10

Minimum Possible Error (CRE_m) 0.189

Expansion Functions

No.	z_1	z_2	z_3
1.	0.000	0.400	0.450
2.	0.400	0.450	0.500
3.	0.450	0.500	0.550
4.	0.500	0.550	0.600
5.	0.550	0.600	1.000
6.			
7.			
8.			
9.			
10.			
11.			
12.			

TEST PROBLEM 8

Datum Relative Error

$E_s(z)$ \ $w_i(z)$	Pulse	Piecewise Sinusoid (Galerkin)	MBCRE
Impulse	0.24	0.29	0.27
Pulse	0.31	0.29	0.30
Triangle	0.31	0.33	0.34

TEST PROBLEM 9

Wire Length 1.0

Radius 0.015

Feed Point 0.5

Gap Width 0.10

Minimum Possible Error (CRE_m) 0.19

Expansion Functions

No.	z_1	z_2	z_3
1.	0.000	0.250	0.500
2.	0.400	0.500	0.600
3.	0.500	0.750	1.000
4.			
5.			
6.			
7.			
8.			
9.			
10.			
11.			
12.			

TEST PROBLEM 9

Datum Relative Error

$E_s(z)$ \ $w_i(z)$	Pulse	Piecewise Sinusoid (Galerkin)	MBCRE
Impulse	0.42	0.22	0.36
Pulse	0.42	0.31	0.32
Triangle	0.54	0.35	0.36

TEST PROBLEM 10

Wire Length 1.0

Radius 0.015

Feed Point 0.500

Gap Width 0.10

Minimum Possible Error (CRE_m) 0.20

Expansion Functions

No.	z_1	z_2	z_3
1.	0.000	0.500	1.000
2.	0.400	0.500	0.600
3.			
4.			
5.			
6.			
7.			
8.			
9.			
10.			
11.			
12.			

TEST PROBLEM 10

Datum Relative Error

$E_s(z)$ \ $w_i(z)$	Pulse	Piecewise Sinusoid (Galerkin)	MBCRE
Impulse	0.22	0.21	0.36
Pulse	0.22	0.31	0.31
Triangle	0.32	0.35	0.36

TEST PROBLEM 11[illegible]

Wire Length 2.0 Radius 0.015

Feed Point 1.0 Gap Width 0.250

Minimum Possible Error (CRE_m) 0.13

Expansion Functions

No.	z_1	z_2	z_3
1.	0.000	0.250	0.500
2.	0.250	0.500	0.750
3.	0.500	0.750	1.000
4.	0.750	1.000	1.250
5.	1.000	1.250	1.500
6.	1.250	1.500	1.750
7.	1.500	1.750	2.000
8.			
9.			
10.			
11.			
12.			

TEST PROBLEM 11

Datum Relative Error

$E_s(z)$ \ $w_i(z)$	Pulse	Piecewise Sinusoid (Galerkin)	MBCRE
Impulse	0.13	0.075	0.75
Pulse	0.13	0.185	0.45
Triangle	0.33	0.284	0.51

```
$      IDENT    BLA00001,PERINI-DB,956700160409,DJB PROGRAM MCP2
$      USERID   BLA00001$DRPR
$      OPTION   FORTRAN
$      FORTY    NLSTIN
C        02/07/75
      SUBROUTINE LINEQ (C,LL)
       COMPLEX C(1),STOR,STO,ST,S
      DIMENSION LR(40)
      DO 20 I=1,LL
   20 LR(I) = I
      M1=0
      DO 18 M=1,LL
      K=M
      DO 2 I=M,LL
      K1=M1*I
      K2=M1*K
      IF(CABS(C(K1))-CABS(C(K2))) 2,2,6
    6 K=I
    2 CONTINUE
      LS=LR(M)
      LR(M)=LR(K)
      LR(K)=LS
      K2=M1*K
      STOR=C(K2)
      J1=0
      DO 7 J=1,LL
      K1=J1+K
      K2=J1*M
      STO=C(K1)
      C(K1)=C(K2)
      C(K2)=STO/STOR
      J1=J1*LL
    7 CONTINUE
      K1=M1*M
       C(K1)=1./STOR
      DO 11 I=1,LL
      IF(I-M) 12,11,12
   12 K1=M1*I
      ST=C(K1)
      C(K1)=0.
      J1=0
      DO 10 J=1,LL
      K1=J1*I
      K2=J1+M
      C(K1)=C(K1)-C(K2)*ST
      J1=J1*LL
   10 CONTINUE
   11 CONTINUE
      M1=M1*LL
   18 CONTINUE
      J1=0
      DO 9 J=1,LL
      IF(J-LR(J)) 14,8,14
   14 LRJ=LR(J)
      J2=(LRJ-1)*LL
   21 DO 13 I=1,LL
      K2=J2*I
      K1=J1*I
      S=C(K2)
      C(K2)=C(K1)
      C(K1)=S
   13 CONTINUE
      LR(J)=LR(LRJ)
      LR(LRJ)=LRJ
      IF(J-LR(J)) 14,8,14
    8 J1=J1*LL
    9 CONTINUE
      RETURN
      END
```

```
$       FORTY    NLSTIN
        COMPLEX FUNCTION EI(X,X1,X2,X3)
        IMPLICIT COMPLEX(C,E,I,V,Y,Z)
        COMPLEX S1,S2,S12
        COMMON Z(1600),ZP(100),ZG(1600),ZOP(100),Y(1600),YP(100),
      1 YG(1600),YOP(100),GL,GH,G1,G2,G3,TE(50),TI(40,3),RAD,
      1 RAD2,B,WL,NE,NC,ID(20),I(40),V(40),RIN,CBJ,NC2,M1,M2,M3,
      1 XX(40),IDS(40),A(20,20),RMIN,ZINO,NPS,KDEL,KDD,ZIN
      1 ,AC(50,20),WLL,WLH
        H1=X2-X1
        H2=X3-X2
        S1=CMPLX(0.,-30./SIN(H1))
        S2=CMPLX(0.,-30./SIN(H2))
        S12=S1*COS(H1)+S2*COS(H2)
        X11=X-X1
        X22=X-X2
        X33=X-X3
        R1=SQRT(X11*X11+RAD2)
        R2=SQRT(X22*X22+RAD2)
        R3=SQRT(X33*X33+RAD2)
        C11=CMPLX(COS(R1),-SIN(R1))/R1
        C22=CMPLX(COS(R2),-SIN(R2))/R2
        C33=CMPLX(COS(R3),-SIN(R3))/R3
        EI=S1*C11-S12*C22+S2*C33
        RETURN
        END
```



```
        FORTY    NLSTIN
        COMPLEX FUNCTION EIC(X,X1,X2,X3)
        IMPLICIT COMPLEX(C,E,I,V,Y,Z)
        COMMON Z(1600),ZP(100),ZG(1600),ZOP(100),Y(1600),YP(100),
      1 YG(1600),YOP(100),GL,GH,G1,G2,G3,TE(50),TI(40,3),RAD,
      1 RAD2,B,WL,NE,NC,ID(20),I(40),V(40),RIN,CBJ,NC2,M1,M2,M3,
      1 XX(40),IDS(40),A(20,20),RMIN,ZINO,NPS,KDEL,KDD,ZIN
      1 ,AC(50,20),WLL,WLH
        EIC=CONJG(EI(X,X1,X2,X3))
        RETURN
        END
```

```
$       FORTY    NLSTIN
        COMPLEX FUNCTION TR(X,X1,X2,X3)
        A=-2./(X3-X1)
        IF(X-X1) 1,,
        IF(X2-X) 2,,
        TR=A*(X-X1)/(X2-X1)
        GO TO 3
      2 IF(X3-X) 1,,
        TR=A*(X3-X)/(X3-X2)
        GO TO 3
      1 TR=0.
      3 RETURN
        END
```

```
        FORTY    NLSTIN
        COMPLEX FUNCTION INPT(F1,A1,A2,A3,F2,B1,B2,B3,XL,XH,NPI)
        IMPLICIT COMPLEX(C,E,I,V,Y,Z)
        COMMON Z(1600),ZP(100),ZG(1600),ZOP(100),Y(1600),YP(100),
      1 YG(1600),YOP(100),GL,GH,G1,G2,G3,TE(50),TI(40,3),RAD,
      1 RAD2,B,WL,NE,NC,ID(20),I(40),V(40),RIN,CBJ,NC2,M1,M2,M3,
      1 XX(40),IDS(40),A(20,20),RMIN,ZINO,NPS,KDEL,KDD,ZIN
      1 ,AC(50,20),WLL,WLH
        COMPLEX F1,F2
        NPT=NPI*(XH-XL)+5
        DELT=(XH-XL)/NPT
        D2=DELT/2
        CT=(0.,0.)
        DO 1 J=1,NPT
        RJ=J-1
        X=XL+RJ*DELT+D2
      1 CT=CT+F1(X,A1,A2,A3)*F2(X,B1,B2,B3)
        INPT=CT*DELT
        RETURN
        END
```

```
        FORTY    NLSTIN
        SUBROUTINE VF(FW,FV,XL,XH)
        IMPLICIT COMPLEX(C,E,I,V,Y,Z)
        COMMON Z(1600),ZP(100),ZG(1600),ZOP(100),Y(1600),YP(100),
      1 YG(1600),YOP(100),GL,GH,G1,G2,G3,TE(50),TI(40,3),RAD,
      1 RAD2,B,WL,NE,NC,ID(20),I(40),V(40),RIN,CBJ,NC2,M1,M2,M3,
      1 XX(40),IDS(40),A(20,20),RMIN,ZINO,NPS,KDEL,KDD,ZIN
      1 ,AC(50,20),WLL,WLH
        COMPLEX FW,FV,PUW,PUV,SP,TR,EI,EIC
        EXTERNAL FW,FV,PUW,PUV,TR,SP,EI,EIC
        DO 1 K=1,NC
        B1=TI(K,1)
        B2=TI(K,2)
        B3=TI(K,3)
      1 V(K)=INPT(FW,B1,B2,B3,FV,G1,G2,G3,XL,XH,15)
        RETURN
        END
```

```
$       FORTY    NLSTIN
        COMPLEX FUNCTION PUW(X,X1,X2,X3)
        A1=(X1+X2)/2.
        A2=(X2+X3)/2.
        PUW=0.
        IF(X-A1) 1,,
        IF(A2-X) 1,,
        PUW=1.
      1 RETURN
        END
```

```
$       FORTY    NLSTIN
        COMPLEX FUNCTION PUV(X,X1,X2,X3)
        COMPLEX PUW
        PUV=2.*PUW(X,X1,X2,X3)/(X3-X1)
        RETURN
        END
```

```
$       FORTY    NLSTIN
        COMPLEX FUNCTION SP(X,X1,X2,X3)
        IMPLICIT COMPLEX(C,E,I,V,Y,Z)
        COMMON Z(1600),ZP(100),ZG(1600),ZOP(100),Y(1600),YP(100),
      1 YG(1600),YOP(100),GL,GH,G1,G2,G3,TE(50),TI(40,3),RAD,
      1 RAD2,B,WL,NE,NC,ID(20),I(40),V(40),RIN,CBJ,NC2,M1,M2,M3,
      1 XX(40),IDS(40),A(20,20),RMIN,ZINO,NPS,KDEL,KDD,ZIN
      1 ,AC(50,20),WLL,WLM
        IF(X-X1) 1,,
        IF(X2-X) 2,,
        SP=SIN(X-X1)/SIN(X2-X1)
        GO TO 3
      2 IF(X3-X) 1,,
        SP=SIN(X3-X)/SIN(X3-X2)
        GO TO 3
      1 SP=0.
      3 RETURN
        END
```

```
$       FORTY   NLSTIN
$       LIMITS   ,30K
        SUBROUTINE SOLVE(L1,L2)
        IMPLICIT COMPLEX(C,E,I,V,Y,Z)
        COMMON Z(1600),ZP(100),ZG(1600),ZOP(100),Y(1600),YP(100),
      1 YG(1600),YOP(100),GL,GH,G1,G2,G3,TE(30),TI(40,3),RAD,
      1 RAD2,B,WL,NE,NC,ID(20),I(40),V(40),RIN,CBJ,NC2,M1,M2,M3,
      1 XX(40),IDS(40),A(20,20),RMIN,ZINO,NPS,KDEL,KDD,ZIN
      1 ,AC(50,20),WLL,WLH
        COMPLEX PUW,PUV,SP,TR,EI,EIC,INPT
        EXTERNAL PUW,PUV,SP,TR,EI,EIC
   42   FORMAT(//,5X,'L1 =',I4,5X,'L2 =',I4,//)
   46   FORMAT(//,5X,'RELATIVE ERROR',3X,E12.4,//)
   53   FORMAT(//,5X,'ZIN =  ',2E12.4,//)
        GO TO (1,2,3), L1
    1   IF(M1) ,,20
        M1=1
        DO 5 K=1,NC
        A1=TI(K,1)
        A2=TI(K,2)
        A3=TI(K,3)
        DO 5 J=1,NC
        B1=TI(J,1)
        B2=TI(J,2)
        B3=TI(J,3)
        D1=(A1+A2)/2.
        D2=(A2+A3)/2.
        KZ=NC*(J-1)+K
        ZP(KZ)=INPT(PUW,A1,A2,A3,EI,B1,B2,B3,D1,D2,15)
    5   YP(KZ)=ZP(KZ)
        CALL LINEQ(YP,NC)
        GO TO 20
    2   IF(M2) ,,20
        M2=1
        DO 6 K=1,NC
        A1=TI(K,1)
        A2=TI(K,2)
        A3=TI(K,3)
        DO 6 J=1,NC
        B1=TI(J,1)
        B2=TI(J,2)
        B3=TI(J,3)
        KZ1=NC*(J-1)+K
        KZ2=NC*(K-1)+J
        ZG(KZ1)=INPT(SP,A1,A2,A3,EI,B1,B2,B3,A1,A3,15)
    6   YG(KZ1)=ZG(KZ1)
        CALL LINEQ(YG,NC)
        GO TO 20
    3   IF(M3) ,,20
        M3=1
        DO 7 K=1,NC
        A1=TI(K,1)
        A2=TI(K,2)
        A3=TI(K,3)
        DO 7 J=1,NC
        B1=TI(J,1)
        B2=TI(J,2)
        B3=TI(J,3)
        KZ1=NC*(J-1)+K
        KZ2=NC*(K-1)+J
        ZOPZ=INPT(EIC,A1,A2,A3,EI,B1,B2,B3,WLL,WLH,15)
        ZOP(KZ1)=ZOPZ
        YOP(KZ1)=ZOPZ
    7   CONTINUE
        CALL LINEQ(YOP,NC)
   20   IF(L1-1) 21,,21
        DO 25 K=1,NC2
        Z(K)=ZP(K)
   25   Y(K)=YP(K)
   21   IF(L1-2) 22,,22
        DO 23 K=1,NC2
        Z(K)=ZG(K)
   23   Y(K)=YG(K)
   22   IF(L1-3) 24,,24
        DO 26 K=1,NC2
        Z(K)=ZOP(K)
   26   Y(K)=YOP(K)
   24   IF(L2-1) 27,,27
        IF(L1-1) 28,,28
        DO 29 K=1,NC
   29   V(K)=-PUW(G2,TI(K,1),TI(K,2),TI(K,3))
   28   IF(L1-2) 30,,30
        DO 31 K=1,NC
   31   V(K)=-SP(G2,TI(K,1),TI(K,2),TI(K,3))
   30   IF(L1-3) 27,,27
        DO 32 K=1,NC
   32   V(K)=-EIC(G2,TI(K,1),TI(K,2),TI(K,3))
   27   IF(L2-2) 33,,33
        IF(L1-1) 34,,34
        CALL VF(PUW,PUV,GL,GH)
   34   IF(L1-2) 35,,35
        CALL VF(SP,PUV,GL,GH)
   35   IF(L1-3) 33,,33
        CALL VF(EIC,PUV,GL,GH)
   33   IF(L2-3) 36,,36
        IF(L1-1) 37,,37
        CALL VF(PUW,TR,G1,G3)
   37   IF(L1-2) 38,,38
        CALL VF(SP,TR,G1,G3)
   38   IF(L1-3) 36,,36
        CALL VF(EIC,TR,G1,G3)
   36   CONTINUE
        WRITE(2,42) L1,L2
        WRITE(6,42) L1,L2
        DO 43 K=1,NC
        CT=(0.,0.)
        DO 44 J=1,NC
        KZ=NC*(J-1)+K
   44   CT=CT+Y(KZ)*V(J)
   43   I(K)=CT
        ZIN=1./I(KDEL)
        WRITE(2,53) ZIN
        WRITE(6,53) ZIN
        IF(KDD=0) ,,52
        NCC=NC
        IF(L2-4) 50,,50
        NCC=NC+1
        DO 51 K=KDEL,NC
        K2=NC+KDEL-K
   51   I(K2+1)=I(K2)
        I(KDEL)=1./ZINO
   50   CT=(0.,0.)
```

```
      DO 45 K=1,NCC
      IK=I(K)-ID(K)
      DO 45 J=1,NCC
45    CT=CT+IK*A(K,J)*CONJG(I(J)-ID(J))
      RER=SQRT(REAL(CT))/RIN
      WRITE(2,46) RER
      WRITE(6,46) RER
      CALL REPT(I,1,NCC,'CURRENTS')
52    RETURN
      END
```

```
      FORTY    WLSTIN
      SUBROUTINE REPT(F1,K1,K2,C)
      COMPLEX F1(1)
      CHARACTER *10C
      WRITE(2,1) C,K1,K2
      WRITE(6,1) C,K1,K2
      DO 2 K=K1,K2
      AR=REAL(F1(K))
      AI=AIMAG(F1(K))
      AM=SQRT(AR*AR+AI*AI)
      IF(AM) ,,3
      AA=0.
      GO TO 4
3     AA=180.*ATAN2(AI,AR)/3.1415926
4     WRITE(2,5) K,AR,AI,AM,AA
2     WRITE(6,5) K,AR,AI,AM,AA
1     FORMAT(//,5X,R10,'  FROM',I4,3X,'TO',I4,//)
5     FORMAT(I5,3E12.4,F9.2)
      RETURN
      END
```



```
$     FORTY    WLSTIN
$     LIMITS   ,30K
      IMPLICIT COMPLEX(C,E,I,V,Y,Z)
      COMMON Z(1600),ZP(100),ZG(1600),ZOP(100),Y(1600),YP(100),
     1 YG(1600),YOP(100),GL,GH,G1,G2,G3,TE(50),TI(40,3),RAD,
     1 RAD2,B,WL,NE,NC,ID(20),I(40),V(40),RIN,CBJ,NC2,M1,M2,M3,
     1 XX(40),IDS(40),A(20,20),RMIN,ZINO,NPS,KDEL,KDD,ZIN
     1 ,AC(50,20),WLL,WLH
      COMPLEX PUW,PUV,SP,TR,EI,EIC,INPT
      EXTERNAL PUW,PUV,SP,TR,EI,EIC
1     FORMAT(5F10.6)
2     FORMAT(10I4)
6     FORMAT(//,5X,'WIRE LENGTH =',F10.6,/,5X,'WAVE NUMBER ='
     1 ,F10.6,/,5X,'RADIUS =',F10.6,//)
8     FORMAT(I5,3F10.6)
9     FORMAT(/7,5X,'GAP  FROM',F10.6,5X,'TO',F10.6,//)
12    FORMAT(//,5X,'FORCED CURRENT',//,I5,3F10.6,//)
16    FORMAT(F10.6,F9.2)
20    FORMAT(3F10.6)
36    FORMAT(5X,'MINIMUM ERROR',3X,E12.4,/,5X,'ZINO',13X,
     1 2E12.4,//)
      M1=0
      M2=0
      M3=0
      KDD=1
      READ(5,1) WL,B,RAD
      B=B*2.*3.1415926
      WL=WL*B
      RAD=RAD*B
      DWD=3.*RAD
      WLL=DWD
      WLH=WL-DWD
      RAD2=RAD*RAD
      READ(5,2) NPS,KPS,KDEL
      DEL=WL/(NPS+1)
      DO 3 K=1,NPS
      TK2=K*DEL
      XX(K)=TK2
      TI(K,1)=TK2-DEL
      TI(K,2)=TK2
3     TI(K,3)=TK2+DEL
      G1=TI(KPS,1)
      G2=TI(KPS,2)
      G3=TI(KPS,3)
      GL=(G1+G2)/2.
      GH=(G2+G3)/2.
      NC=NPS
      NC2=NC*NC
      CALL SOLVE(2,1)
```

```
      DO 31 K=1,NPS
   31 IDS(K)=I(K)
      CALL REPT(IDS,1,NPS,'STANDARD I')
      M1=0
      M2=0
      M3=0
      READ(5,2) NC,NE
      NC2=NC*NC
      DO 33 K=1,NE
   33 READ(5,1) TE(K)
      DO 5 K=1,NC
      READ(5,2) K1,K2,K3
      TI(K,1)=B*TE(K1)
      TI(K,2)=B*TE(K2)
    5 TI(K,3)=B*TE(K3)
      READ(5,2) KL,KH
      DO 22 K=1,NC
      A1=TI(K,1)
      A2=TI(K,2)
      A3=TI(K,3)
      CT=(0.,0.)
      DO 21 KI=1,NPS
      X1=XX(KI)-DEL
      X2=XX(KI)
      X3=XX(KI)+DEL
      AC(KI,K)=INPT(SP,A1,A2,A3,SP,X1,X2,X3,X1,X3,15)
   21 CT=CT+AC(KI,K)*IDS(KI)
   22 V(K)=CT
      DO 32 K=1,NC
      A1=TI(K,1)
      A2=TI(K,2)
      A3=TI(K,3)
      DO 32 J=K,NC
      B1=TI(J,1)
      B2=TI(J,2)
      B3=TI(J,3)
      KZ1=NC*(J-1)+K
      KZ2=NC*(K-1)+J
      AA=INPT(SP,A1,A2,A3,SP,B1,B2,B3,A1,A3,15)
      A(K,J)=AA
      A(J,K)=AA
      Y(KZ1)=AA
   32 Y(KZ2)=AA
      CALL LINEQ(Y,NC)
      DO 24 K=1,NC
      CT=(0.,0.)
      DO 25 J=1,NC
      KZ=NC*(J-1)+K
   25 CT=CT+Y(KZ)*V(J)
   24 ID(K)=CT
      ZINO=1./ID(KDEL)
      CT=(0.,0.)
      DO 26 K=1,NC
      IDK=ID(K)
      DO 26 J=1,NC
   26 CT=CT+IDK*A(K,J)*CONJG(ID(J))
      RIN=SQRT(REAL(CT))
      KDD=0
      A1=0.
      A2=DEL
      A3=A2+DEL
      A4=A3+DEL
      P11=INPT(SP,A1,A2,A3,SP,A1,A2,A3,A1,A3,15)
      P12=INPT(SP,A1,A2,A3,SP,A2,A3,A4,A1,A3,15)
      CT1=IDS(1)*CONJG(IDS(1))
      CT2=(0.,0.)
      DO 37 K=2,NPS
      CT1=CT1+IDS(K)*CONJG(IDS(K))
   37 CT2=CT2+IDS(K)*CONJG(IDS(K-1))
      R11=P11*CT1+P12*2.*REAL(CT2)
      CT=(0.,0.)
      DO 38 K=1,NPS
      IK=IDS(K)
      DO 38 J=1,NC
   38 CT=CT+IK*AC(K,J)*CONJG(ID(J))
      R12=CT
      RMIN=SQRT(R11+2.*R12+RIN*RIN)/RIN
      GL=B*TE(KL)
      GH=B*TE(KH)
      GC=(GL+GH)/2.
      GD=GH-GL
      G1=GC-GD
      G2=GC
      G3=GC+GD
      WRITE(2,6) WL/B,B,RAD/B
      WRITE(6,6) WL/B,B,RAD/B
      DO 7 K=1,NC
      WRITE(2,8) K,TI(K,1)/B,TI(K,2)/B,TI(K,3)/B
    7 WRITE(6,8) K,TI(K,1)/B,TI(K,2)/B,TI(K,3)/B
      WRITE(2,9) GL/B,GH/B
      WRITE(6,9) GL/B,GH/B
      WRITE(2,36) RMIN,ZINO
      WRITE(6,36) RMIN,ZINO
      CALL REPT(ID,1,NC,'DATUM I')
      DO 10 K=1,3
      DO 10 KK=1,3
   10 CALL SOLVE(K,KK)
      STOP
      END
```